

Friedmann inflation in Horava-Lifshitz gravity with a scalar field

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We study Friedmann inflation in general Horava-Lifshitz (HL) gravity with detailed and non-detailed but also without the projectability conditions. Accordingly, we derive the modifications in the Friedmann equations due to single scalar field potentials describing power-law and minimal-supersymmetrically extended inflation. By implementing four types of the equations-of-state characterizing the cosmic background geometry, the dependence of the tensorial and spectral density fluctuations and their ratio on the inflation field is determined. The latter characterizes the time evolution of the inflation field relative to the Hubble parameter. Furthermore, the ratio of tensorial-to-spectral density fluctuations is calculated in dependence on the spectral index. The resulting slow-roll parameters apparently differ from the ones deduced from the standard General Relativity (Friedmann gravity). We also observe that the tensorial-to-spectral density fluctuations continuously decrease when moving from non-detailed HL gravity, to Friedmann gravity, to HL gravity without the projectability, and to detailed HL gravity. This regular pattern is valid for three types of cosmic equations-of-state and different inflation potential models. The results fit well with the recent PLANCK observations.

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I. INTRODUCTION

It is widely believed that the Universe in its very early stages went through a rapid exponential expansion [5]. At the beginning of this inflation epoch, scales inside the Hubble radius expanded, exponentially, as well [37, 39]. At a later time, due to large-scale cosmological perturbations with a scale-invariant spectrum, they reentered the Hubble radius. This scenario on the inflation era [3, 17, 30] disfavors exotic relics such as primordial magnetic monopoles [31], which otherwise would alter the present Universe. Also, it was assumed that the exponential expansion could eliminate the particle horizon and therefore helps in solving the horizon problem [23]. Furthermore, the inflation scenario gives a solid explanation for flatness and homogeneity of our Universe and evolves quantum fluctuations in classical curvature perturbation, as well [15, 33]. It is conjectured that during the inflation era the perturbations started out with a small amplitude and gradually grown. This leads to a structure formation and thus even would explain the large-scale structure of the Universe. While detailed mechanism responsible for inflation is not well known yet, a number of predictions have been confirmed by observations [41, 43] and the hypothetical field inflaton [18] is thought as a likely candidate.

According to the cosmological equations suggested by the non-relativistic renormalizable gravity [10, 11], the early Universe gets features that may give an alternative to the inflation. The resulting theory, known as Horava-Lifshitz (HL) gravity would be able to avoid the Big Bang singularity [11]. Quantum corrections to gravity are conjectured to replace this singularity with an exponentially expanding de Sitter phase [40]. Recently, the HL gravity is assumed to introduce possible modifications in the standard General Relativity (GR) gravity, especially in strong gravitational regimes [19, 29]. This is accompanied by modifications in the gravity itself making it responsible for the accelerated expansion of the Universe. Therefore, one of the natural implications of HL gravity is on the very early Universe. Furthermore, various modified theories of gravity, which are successfully able to unify different inflationary approaches with the late-time Universe acceleration and recent cosmological observations, can be implemented [13, 35].

It was demonstrated that the early-time inflation of the FRW Universe might take place as a power-law accelerating evolution or similar to de Sitter expansion [9]. The earlier shall have a future singularity called sudden or Big Rip. When taking into consideration covariant gravity with a scalar field by an extra higher derivative term, which preserves the ultraviolet behavior of the graviton propagator, the future singularity can be cured and the Einstein frame-action [8], where $f(R) \sim f_0 R^\alpha$, with constant f_0 and α should be modified, accordingly. Furthermore, the presence of R^2 -term in the consistent modified gravity removes the future singularity [8]. The emergence of finite-time future singularities has been studied in Ref. [16]. It was found that such singularities can be cured by adding a higher-order spatial derivative term. Recently, the general formulas for the inflationary power spectra of scalar and tensor are driven in the presence of a scalar field [22].

Based HL gravity without an additional scalar degree-of-freedom, a simple scenario for the scale-invariant quantum-fluctuations was proposed [36] in order to impose inflation. For this simple scenario, the detailed balance conditions were not necessary but the inflation scenario itself may be still or no longer needed, for instance, serious horizon problems remain unsolved, such as monopole and domain walls. They still require inflation with slow-roll conditions.

In HL gravity without the projectability condition, the inflation regime was studied [47]. But, opposite to the proposal of Ref. [36], the linear scalar perturbations equations of the FLRW universe are derived for a single scalar field. A master equation of the perturbations has been specified for a particular gauge. The power spectrum and spectrum index of the comoving curvature perturbations have been determined. It was noticed that the perturbations remain scale-invariance, and the HL gravity without the projectability condition is consistent with all current cosmological observations. This another solid support for adding scalar field(s).

In framework of nonrelativistic HL gravity with the projectability condition and an arbitrary coupling constant (λ), the inflation was studied [21]. Accordingly, the FLRW Universe without specifying the gauge is necessarily flat. But by adding a single scalar field, it was noticed that both metric and scalar field become strongly coupled and almost identically oscillating in sub-horizon regions. In super-horizon regions, the comoving curvature perturbation remains constant although the FLRW perturbations become adiabatic. Furthermore, the perturbations in the slow-roll parameters, for instance, both scalar and tensor are found scale-invariant. Concrete tuning the coupling coefficients makes the spectrum index of the tensor perturbation identical as the ones deduced from GR. But the ratio of scalar to tensor spectra can be similar to that from GR and seems to depend on the spatial higher-order derivative terms.

Following this line, we apply two models for the cosmic inflation and HL gravity. It intends to find out consistency with the recent cosmological observations and the possible distinguishability from GR gravity. We shall compare between the various corrections proposed to reduce the independently coupling constants of the

HL gravity theory. In section II, a short reminder to Horava-Lifshitz gravity shall be outlined. The modified Friedmann equations due to HL gravity are studied under non-detailed, section II A, and detailed balance conditions, section II B and that without the projectability condition, section II C. The cosmic inflation models are implemented in section III A. Section III B is devoted to study the fluctuations and the slow-roll parameters in HL gravity. The results shall be confronted to the recent PLANCK observations in section III C. Section IV is devoted to final remarks and conclusions.

II. HORAVA-LIFSHITZ GRAVITY

Starting from the dynamical variables, the lapse and shift vectors, N and N_i , respectively, and canonical gravity, the full space-time metric reads

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (1)$$

where the indices are raised and lowered using the spatial metric g_{ij} . To explain quantum critical phenomena in condensed matter, Lifshitz proposed a scalar field theory [29]. This has inspired Horava to propose a quantum gravity theory with an anisotropic scaling in ultraviolet (UV) [19]. Anisotropic UV scaling relations between Minkowskian space and time are thus the basic assumptions of this theory. In other words, the quantum gravity scaling at short distances exhibits a strong anisotropy between space and time

$$t \longrightarrow l^z t, \quad x^i \longrightarrow l x^i, \quad (2)$$

where z is the dynamical critical exponent given in ultraviolet, and l is a constant performing the scaling. As anisotropic scaling apparently implies a preferred time coordinate, 4-dimensional Lorentzian metric can not be the only fundamental structure on Lorentzian (pseudo-Riemannian) manifold. In order to single out spatial (temporal) coordinates in a differentiable manifold, it was assumed that codimension foliation as a basic structure on this manifold is resorted [19]. The foliation structure in turn fulfils local Galilean invariance but also implies impossible full-diffeomorphism invariance in GR. This is recovered in some limits of the critical exponent z . Thus, GR is considered as an emerging in an infrared fixed point [24]. To summarize, HL gravity is a projectable approach minimizing the number of independent couplings in potential and adopting an extra principle to build the potential, i.e. detailed balance condition.

The detailed balance condition is a technical treatment to further reduce the coupling constants, which would limit the prediction powers of the HL gravity theory. Furthermore, it was argued that the detailed balance HL gravity has to be broken in order to enable the theory to be compatible with the available observations [14, 20]. A possible connection between detailed balance conditions and the entropic origin of gravity was discussed in Ref. [44]. Therefore, in constructing the potential, the detailed balance HL gravity can be implemented instead of effective field theory.

The non-detailed balance condition was also proposed [19] in order to reduce the number of independently coupling constants [6, 7, 25]. These are likely when general covariance is abandoned [47]. This condition assumes that the gravitational potential can be obtained from a superpotential defined on the three-spatial hypersurfaces $t = \text{const}$, the Newtonian limit would not exist [34], and a scalar field in UV could not be stable [12].

The projectability condition, which is usually taken into consideration with the balance conditions, assumes that the lapse function in the Arnowitt-Deser-Misner (ADM) decompositions is only time dependent. Recently, HL gravity without the projectability condition has been introduced [47].

In HL gravity, the general structure of the scalar field action is given by a consistent imposing of the corresponding symmetries [10, 11]. However, one can in the construction of the action obtain the scalar field,

$$\mathcal{S}^\phi = \int dt d^3x \sqrt{g} N \left[\frac{(3\lambda - 1)\dot{\phi}^2}{4N^2} + m_1 m_2 \phi \nabla^2 \phi - \frac{1}{2} m_2^2 \phi \nabla^4 \phi + \frac{1}{2} m_3^2 \phi \nabla^6 \phi - V(\phi) \right], \quad (3)$$

where m_i are constants and $V(\phi)$ plays the role of an inflation potential.

A. Non-detailed balance HL gravity

We shortly introduce the modified Friedmann equations in non-detailed balance HL gravity [42]

$$H^2 = \frac{\kappa^2}{6(3\lambda-1)}\rho - \frac{\kappa^4\mu^2\Lambda_\omega^2}{16(3\lambda-1)^2} + \frac{\kappa^4 K \mu^2 \Lambda_\omega}{8(3\lambda-1)^2 a^2} - \frac{\kappa^4 K^2 \mu^2}{16(3\lambda-1)^2 a^4}, \quad (4)$$

$$\dot{H} + \frac{3}{2}H^2 = -\frac{\kappa^2}{4(3\lambda-1)}p - \frac{\kappa^4\mu^2\Lambda_\omega^2}{32(3\lambda-1)^2} + \frac{\kappa^4 K \mu^2 \Lambda_\omega}{16(3\lambda-1)^2 a^2}, \quad (5)$$

where Hubble parameter $H = \dot{a}/a$, with $\dot{H} = \ddot{a}/a - H^2$. When we let $\xi = \kappa^2/(3\lambda-1)$, Eq. (4) becomes

$$H^2 = \frac{\xi}{6}\rho - \frac{\xi^2\mu^2}{16} \left[\Lambda_\omega^2 - \frac{2K\Lambda_\omega}{a^2} + \frac{K^2}{a^4} \right]. \quad (6)$$

From first law of thermodynamics, the continuity equation can be given as

$$\rho = \rho_o a^{-3(1+\omega)}, \quad (7)$$

which equivalently leads to $a \approx \rho^{-1/[3(1+\omega)]}$. Then Eq. (6) can be written as

$$H^2 = \frac{\xi}{6}\rho - \frac{\xi^2\mu^2}{16} \left[\Lambda_\omega^2 - 2K\Lambda_\omega \rho^{\frac{2}{3(1+\omega)}} + K^2 \rho^{\frac{4}{3(1+\omega)}} \right]. \quad (8)$$

From classical field theory, the scalar field (ϕ) in the total inflation energy is assumed to couple with the gravity

$$\frac{1}{2} \left(\dot{\phi}^2 + (\nabla\phi)^2 \right) + V(\phi). \quad (9)$$

Motivation of introducing such scalar field was shortly detailed in introduction. Furthermore, the inflation dynamics can be described by various equations, for instance,

- Friedmann equation, which describes the contraction and expansion of the Universe,

$$H^2 + \frac{\kappa c^2}{a^2} = \frac{8\pi G \rho + \Lambda c^2}{3}, \quad (10)$$

especialy in homogeneous and isotropic Universe, and

- Klein-Gordon equation, which is the simplest equation of motion for a spatially homogeneous scalar field

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = 0, \quad (11)$$

where $\partial_\phi \equiv \partial/\partial\phi$.

The energy density sums up inflation and cosmological constant contributions, i.e. $\rho = \rho_\phi + \rho_v$. Then, the modified Friedmann equation becomes

$$\begin{aligned} H^2 = & \frac{\xi}{6} \left[\frac{\dot{\phi}^2}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2} + \rho_v \right] \\ & - \frac{\xi^2\mu^2}{16} \left\{ \Lambda_\omega^2 - 2K\Lambda_\omega \left(\frac{\dot{\phi}^2}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2} + \rho_v \right)^{\frac{2}{3(1+\omega)}} \right. \\ & \left. + K^2 \left(\frac{\dot{\phi}^2}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2} + \rho_v \right)^{\frac{4}{3(1+\omega)}} \right\}, \end{aligned} \quad (12)$$

where $\rho_v = \Lambda\omega/8\pi G$ with G being the gravitational constant. In rapidly expanding Universe and if the inflation field starts out sufficiently homogeneously, the inflation field becomes minimum, i.e. very slow [27, 32].

This would be modelled by a sphere in a viscous medium, where both energy densities due to matter (ρ_m) and radiation (ρ_r) are neglected

$$(\nabla\phi)^2 \ll V(\phi), \quad (13)$$

$$\ddot{\phi} \ll 3H\dot{\phi}, \quad (14)$$

$$\dot{\phi}^2 \ll V(\phi). \quad (15)$$

The first inequality, Eq. (13), is obtained under the assumption of homogeneity and isotropy of Friedmann-Lemaitre-Robertson-Walker (FLRW) Universe [27, 32], while the second inequality, Eq. (14), states that the scalar field changes very slowly so that the acceleration could be neglected [27, 32]. The third inequality, Eq. (15), gives a principle condition for the Universe expansion. Accordingly, the kinetic energy is much less than the potential energy [27, 32]. But it was observed that the Universe expansion apparently accelerates [30]. Therefore, the modified Friedmann equation becomes

$$H^2 = \frac{\xi}{6} \left[V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right] - \frac{\xi^2 \mu^2}{16} \left[\Lambda_\omega^2 + 2K\Lambda_\omega \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^{\frac{2}{3(1+\omega)}} + K^2 \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^{\frac{4}{3(1+\omega)}} \right], \quad (16)$$

which obviously depends on the equation of state:

- for dust approximation, i.e. $\omega = 0$:

The corresponding modified Friedmann equation, Eq. (16), can be given as

$$H^2 = \frac{\xi}{6} \left[V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right] - \frac{\xi^2 \mu^2}{16} \left[\Lambda_\omega^2 - 2K\Lambda_\omega \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^{\frac{2}{3}} + K^2 \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^{\frac{4}{3}} \right], \quad (17)$$

- for matter-dominated era, i.e. $\omega = 1/3$

$$H^2 = \frac{\xi}{6} \left[V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right] - \frac{\xi^2 \mu^2}{16} \left[\Lambda_\omega^2 - 2K\Lambda_\omega \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^{\frac{1}{2}} + K^2 \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right) \right], \quad (18)$$

- for cold dark matter-dominated era, i.e. $\omega = -1/3$

$$H^2 = \frac{\xi}{6} \left[V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right] - \frac{\xi^2 \mu^2}{16} \left[\mu\Lambda_\omega^2 - 2K\Lambda_\omega \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right) + K^2 \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^2 \right], \quad (19)$$

- and for non-vanishing cosmological constant, i.e. $\omega = -1$

$$H^2 \longrightarrow \infty. \quad (20)$$

B. Detailed balance HL gravity

The modified Friedmann equations due to detailed balance HL gravity is given as [42]

$$H^2 = \frac{2}{(3\lambda - 1)} \left[\frac{\Lambda_\omega}{2} + \frac{8\pi G_N}{3} \rho - \kappa a^{-2} + \frac{\kappa^2}{2\Lambda_\omega} a^{-4} \right], \quad (21)$$

which reads the continuity equation,

$$H^2 = \frac{2}{(3\lambda - 1)} \left[\frac{\Lambda_\omega}{2} + \frac{8\pi G_N}{3} \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right) - \kappa \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^{\frac{2}{3(1+\omega)}} + \frac{k^2}{2\Lambda_\omega} \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^{\frac{4}{3(1+\omega)}} \right]. \quad (22)$$

The various equations of state lead to

- for dust approximation, i.e. $\omega = 0$:

The modified Friedmann equation, Eq. (22), reads

$$H^2 = \frac{2}{(3\lambda - 1)} \left[\frac{\Lambda_\omega}{2} + \frac{8\pi G_N}{3} \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right) - \kappa \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^{\frac{2}{3}} + \frac{k^2}{2\Lambda_\omega} \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^{\frac{4}{3}} \right], \quad (23)$$

- for matter-dominated era, i.e. $\omega = 1/3$

$$H^2 = \frac{2}{(3\lambda - 1)} \left[\frac{\Lambda_\omega}{2} + \frac{8\pi G_N}{3} \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right) - \kappa \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^{\frac{1}{2}} + \frac{k^2}{2\Lambda_\omega} \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right) \right], \quad (24)$$

- for cold dark matter-dominated era, i.e. $\omega = -1/3$

$$H^2 = \frac{2}{(3\lambda - 1)} \left[\frac{\Lambda_\omega}{2} + \frac{8\pi G_N}{3} \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right) - \kappa \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right) + \frac{k^2}{2\Lambda_\omega} \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right)^2 \right], \quad (25)$$

- and for non-vanishing cosmological constant, i.e. $\omega = -1$

$$H^2 \longrightarrow \infty. \quad (26)$$

C. HL gravity without the projectability condition

Both detailed and non-detailed corrections remarkably reduce the coupling constants. But essential problems in the HL gravity theory remain, such as strong coupling problem exists. Due to the existence of spin-0 graviton, Minkowski spacetime remains unstable, despite the de Sitter spacetime does. Spin-0 graviton causes difference of its speed from that of spin-2 graviton. Both are not related by any symmetry. Thus, any attempt to restore Lorentz symmetry at low energies is a big challenge, i.e. no mechanism ensuring that all species of matter and gravity have the same effective speed. Such problems have been discussed in Ref. [47]. HL gravity without the projectability condition was proposed to solve these problems. It assumes that the detailed balance condition is softly broken, and the symmetry of the HL gravity theory is thus enlarged to include local U(1) symmetry [45, 46]. The latter eliminates spin-0 graviton. In flat FLRW space-time, the coupling constant Λ_g vanishes [47], despite its radiative corrections expected from quantum mechanics. Then at vanishing cosmological constant,

$$ds^2 = a^2(\eta) [-d\eta^2 + \gamma_{ij} dx^i dx^j], \quad (27)$$

where $\eta = \int (1/a) dt$ is the conformal time, and $\gamma_{ij} = \delta_{ij}/(1 + \kappa r^2/4)$. The modified Friedmann equation is

$$'H^2 = \frac{8\pi \tilde{G} a^2}{3} \left(\frac{1}{2} \hat{\phi}'^2 + \tilde{V}(\hat{\phi}) \right), \quad (28)$$

where $'H = H a$, $'\phi = \dot{\phi} a$, $\tilde{G} = 2 f G/(3\lambda - 1)$, the background scalar field $\hat{\phi} = \phi(\eta)$ and inflation potential $\tilde{V}(\hat{\phi}) = V(\phi)/f(\lambda)$

$$H^2 = \frac{16\pi f G}{3(3\lambda - 1)} \left(V(\phi) + \frac{\Lambda_\omega}{8\pi G} \right). \quad (29)$$

The right-hand side include energy densities related to the scalar field and to the cosmological constant. Thus, Eq. (29) does not depend on equation of state (EoS). It is obvious that Eq. (29) can be reduced to the standard Friedmann equation, i.e. GR gravity, at $f = \lambda = 1$.

III. RESULTS

A. Cosmic inflation models

We propose to implement two models for the inflation potential:

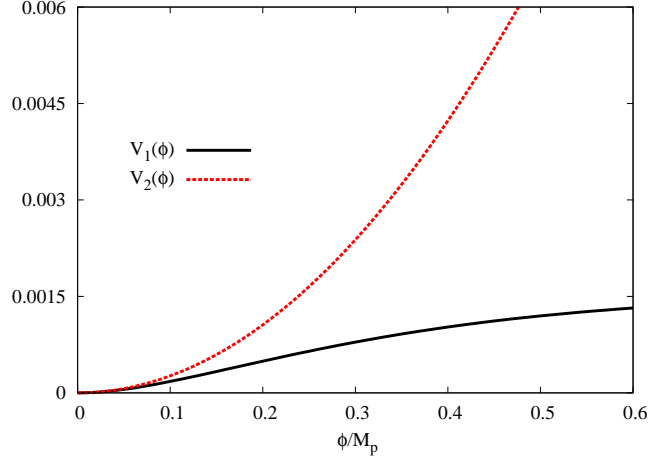


Fig. 1: The hybrid inflation potential $V(\phi)$ with quadratic- [Eqs. (30) solid curve] and higher-term [(32) dashed curve] is given in dependence on varying inflation field ϕ/M_{pl} .

1. for a power-law inflation with the free parameter d , an inflation potential is given as [26, 40]

$$V_1(\phi) = \frac{3M_{pl}^2 d^2}{32\pi} \left[1 - \exp\left(-\frac{16\pi^{1/2}}{3M_{pl}^2} \phi\right) \right]^2, \quad (30)$$

2. and another inflation potential is based on certain minimal supersymmetric extensions of the standard model for elementary particles [4]

$$V_2(\phi) = \left(\frac{m^2}{2}\right) \phi^2 - \left(\frac{\sqrt{2\lambda(n-1)}m}{n}\right) \phi^n + \left(\frac{\lambda}{4}\right) \phi^{2(n-1)}. \quad (31)$$

The effects of this scalar potential have been studied, recently [4, 38]. There are two free parameters, m and λ , while $n > 2$ is an integer. At $n = 3$,

$$V_2(\phi) = \left(\frac{m^2}{2}\right) \phi^2 - \left(\frac{2\sqrt{\lambda}m}{3}\right) \phi^3 + \left(\frac{\lambda}{4}\right) \phi^4, \quad (32)$$

This approximation shall be utilized in the present calculations.

The dependence of the hybrid inflation with quadratic- and higher-term potential ($V(\phi)$) on varying inflation field (ϕ/M_{pl}), Eqs. (30) and (32), respectively, is illustrated in Fig. 1. We find that $V_1(\phi)$ slowly increases with increasing ϕ/M_{pl} (solid curve). It can approach a saturation at large ϕ/M_{pl} , while $V_2(\phi)$ shows a rapid and apparently steady increase (dashed curve). Bearing in mind these differences, it intends to compare between the impacts of these scalar fields on the inflationary regime.

B. Fluctuations and Slow-Roll Parameters

In very early Universe, the scalar field (ϕ) is assumed to derive the inflation [26, 28, 30]. As introduced in Ref. [36], it was believed that the detailed balance conditions for HL gravity were not necessary for the inflation. But, serious horizon problems, such as monopole and domain walls, remain unsolved. The inclusion of scalar fields is essential to generate inflation even in HL gravity. The main slow-roll parameters are given as

$$\epsilon_H \equiv \frac{M_{pl}^2}{16\pi} \left(\frac{\partial_\phi V(\phi)}{V(\phi)} \right)^2, \quad (33)$$

$$\eta_H \equiv \frac{M_{pl}^2}{8\pi} \left(\frac{\partial_\phi^2 V(\phi)}{V(\phi)} \right), \quad (34)$$

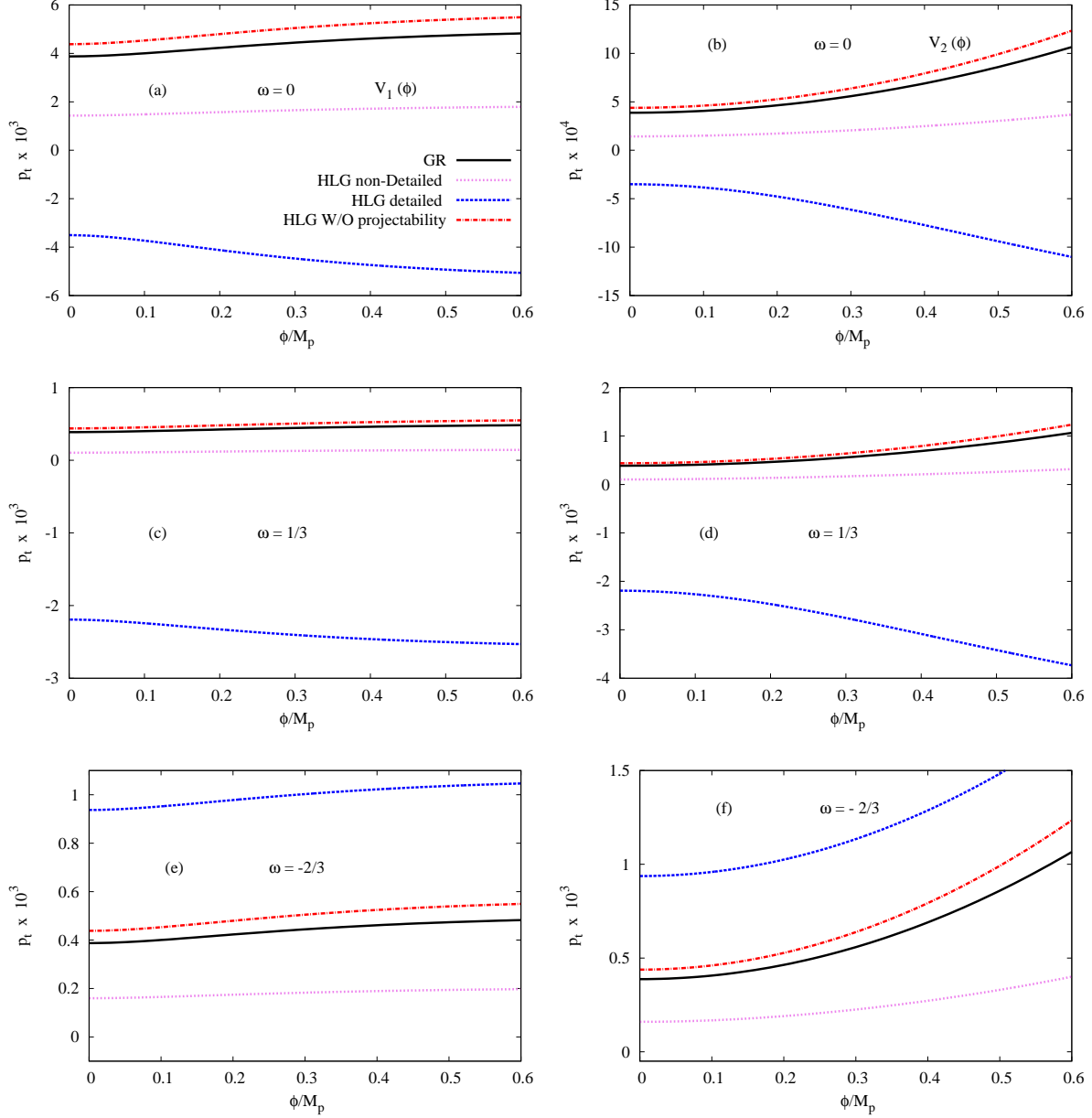


Fig. 2: The tensorial density fluctuations (P_t) from GR (solid curve), HL gravity detailed (dashed curve), HL gravity non-detailed (dotted curve) and HL gravity without the projectability (dash-dotted curve) are given as functions of the inflation field (ϕ) in units of Planck mass M_p for two inflation potentials and various EoS; $\omega = 0$ (top panel), $1/3$ (middle panel), and $-2/3$ (bottom panel).

where $V(\phi)$ can be any inflation potential such as either Eq. (30) or (32). The main reason why this scalar field or another one is chosen is obviously merely motivated by its ability to cope with trusted observations. We shall judge about the different fields due their abilities to reproduce the recent PLANCK observations. We utilize different equations of state characterizing the cosmic background geometry.

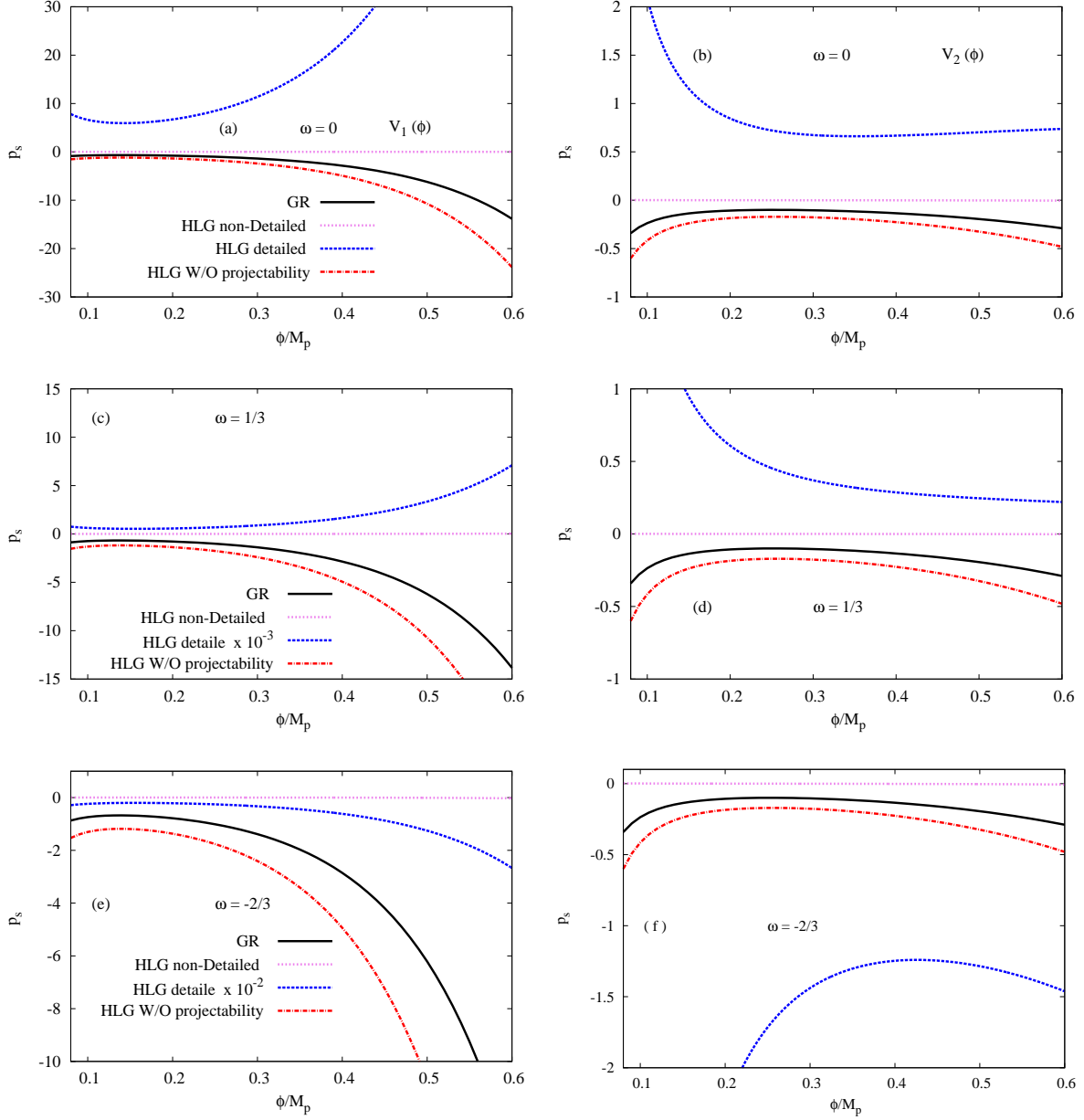


Fig. 3: The same as in Fig. 2 but for spectral density fluctuations (P_s).

The tensorial and scalar density fluctuations, respectively, can be given as [26, 28, 30]

$$P_t = \left(\frac{H}{2\pi}\right)^2 \left[1 - \frac{H}{\Lambda} \sin\left(\frac{2\Lambda}{H}\right)\right], \quad (35)$$

$$P_s = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 \left[1 - \frac{H}{\Lambda} \sin\left(\frac{2\Lambda}{H}\right)\right]. \quad (36)$$

Instead, it is more convenient to study the ratio of tensor-to-scalar fluctuations [26, 28, 30]

$$r = \frac{P_t}{P_s} = \left(\frac{\dot{\phi}}{H}\right)^2, \quad (37)$$

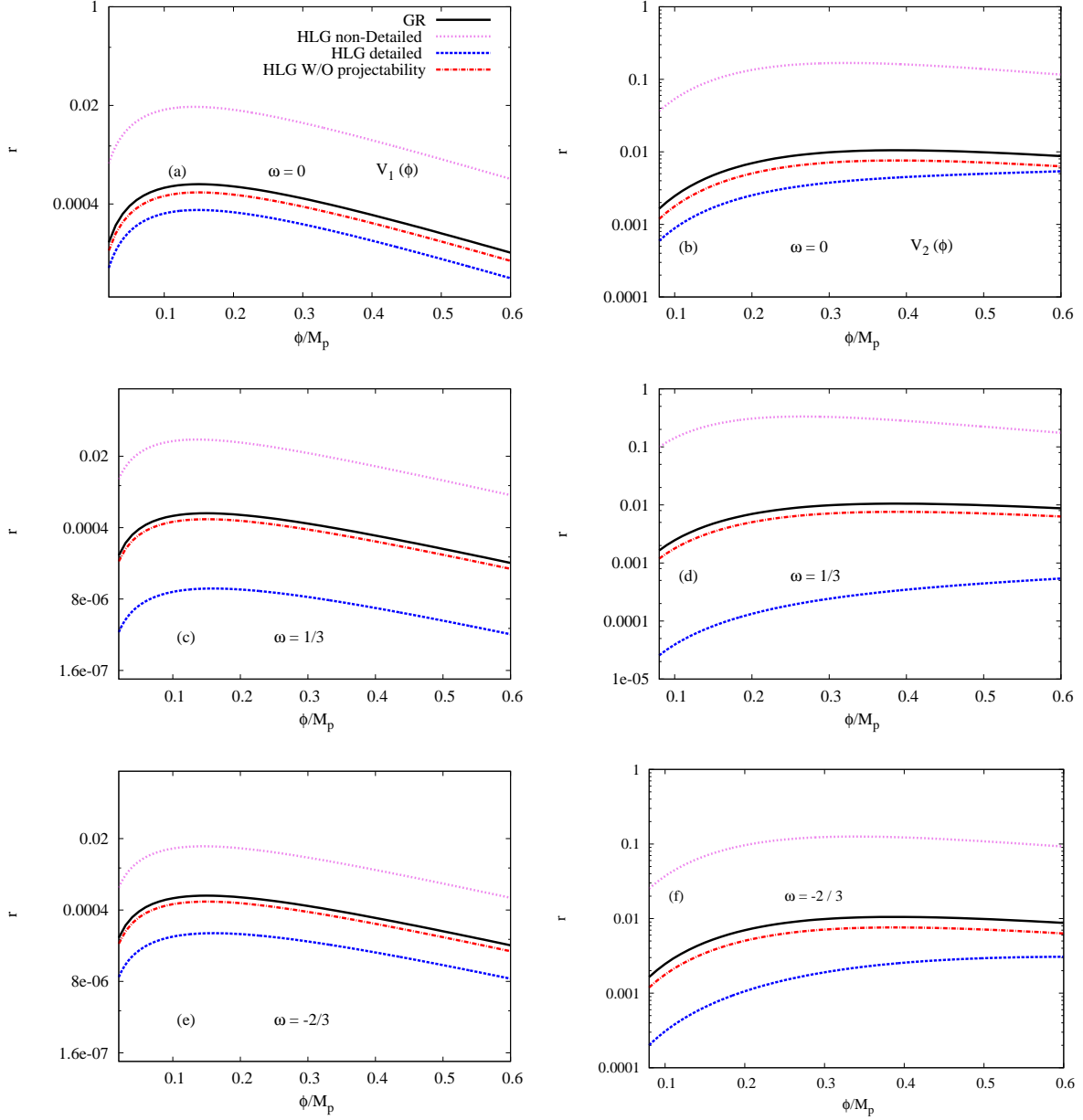


Fig. 4: The same as Fig. 2 but for tensorial-to-scalar density fluctuations ($r = P_t/P_s$).

where the dependence on many parameters can be eliminated. The time derivations of the inflation field are

$$\dot{\phi} = \frac{-1}{3H} \partial_{\phi} V(\phi), \quad (38)$$

$$\ddot{\phi} = \frac{-\dot{\phi}}{3H} \partial_{\phi}^2 V(\phi). \quad (39)$$

The most interesting aspects of the inflation scenario are the predictions of the quantum perturbation. Apart from employing the amplitude of scalar perturbations, we define the scalar spectra index

$$n_s \equiv 1 - \sqrt{\frac{r}{3}}. \quad (40)$$

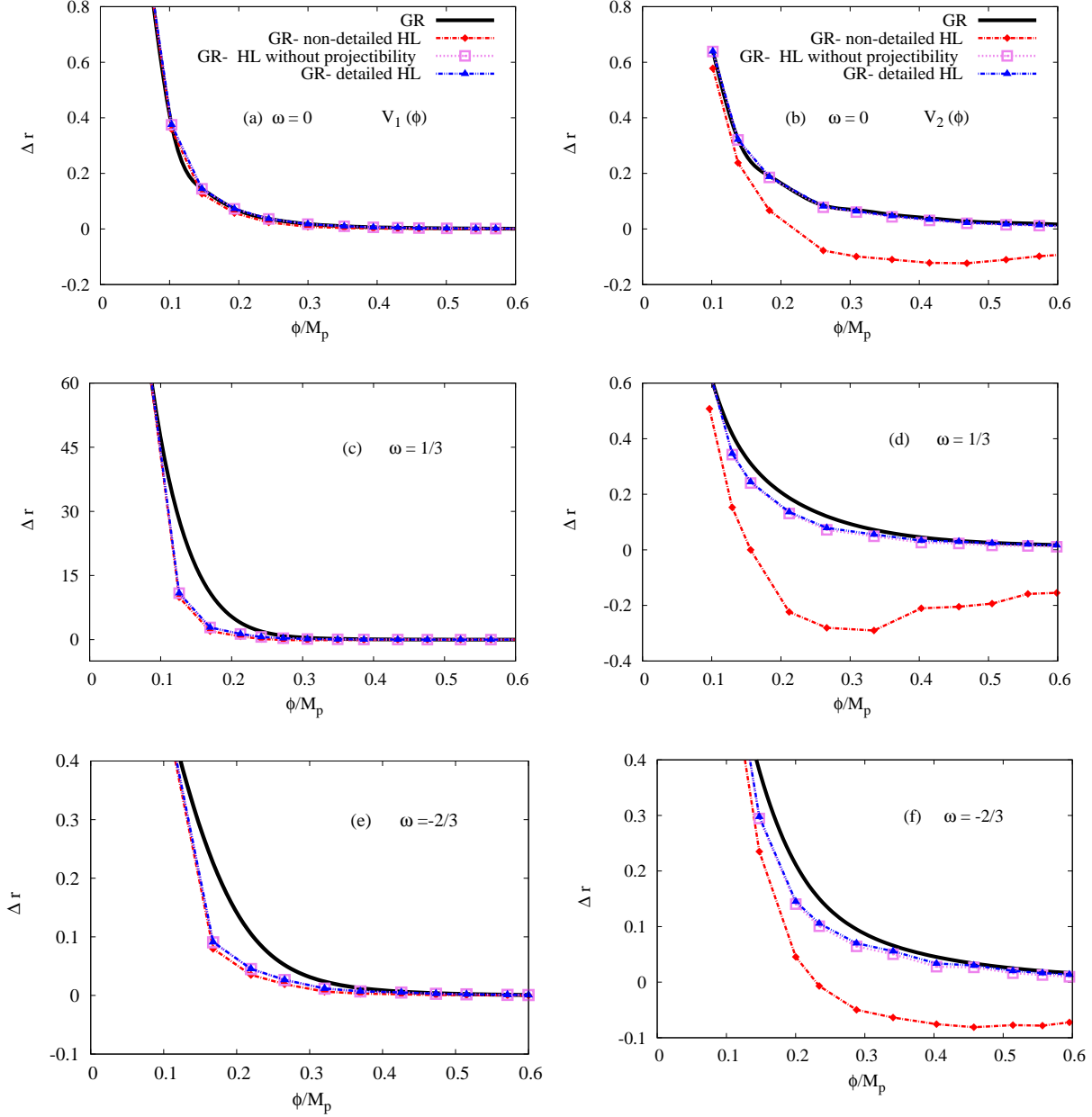


Fig. 5: The same as Fig. 2 but here the net GR-HL impacts [dashed-lines with diamonds (non-detailed), dotted lines with squares (without projectibility) and double-dotted dashed lines with triangles (detailed)] to the inflation parameter r , Eq. (37), are compared with that from pure GR gravity (solid curve).

In Fig. 2, the tensorial density fluctuations (P_t) are given as functions of the inflation field (ϕ) in units of Planck mass (M_p), Eq. (35), for two inflation potentials, Eqs. (30) and (32), and different EoS; $\omega = 0$ (top panel), $1/3$ (middle panel), and $-2/3$ (bottom panel). The calculations are performed in GR (solid curve), HL gravity detailed (dashed curve), HL gravity non-detailed (dotted curve) and HL gravity without the projectibility (dash-dotted curve) condition and implementing two types of inflation potentials; Eq. (30) in left- and (32) in right-hand panel.

The GR results are almost equivalent to HL gravity without the projectibility condition. Also we notice that $V_1(\phi)$ results in values smaller than that from $V_2(\phi)$. P_t from detailed HL gravity is the smallest, except at $\omega = -2/3$, it becomes the largest. P_t from non-detailed HL gravity with is always smaller than GR (Friedmann

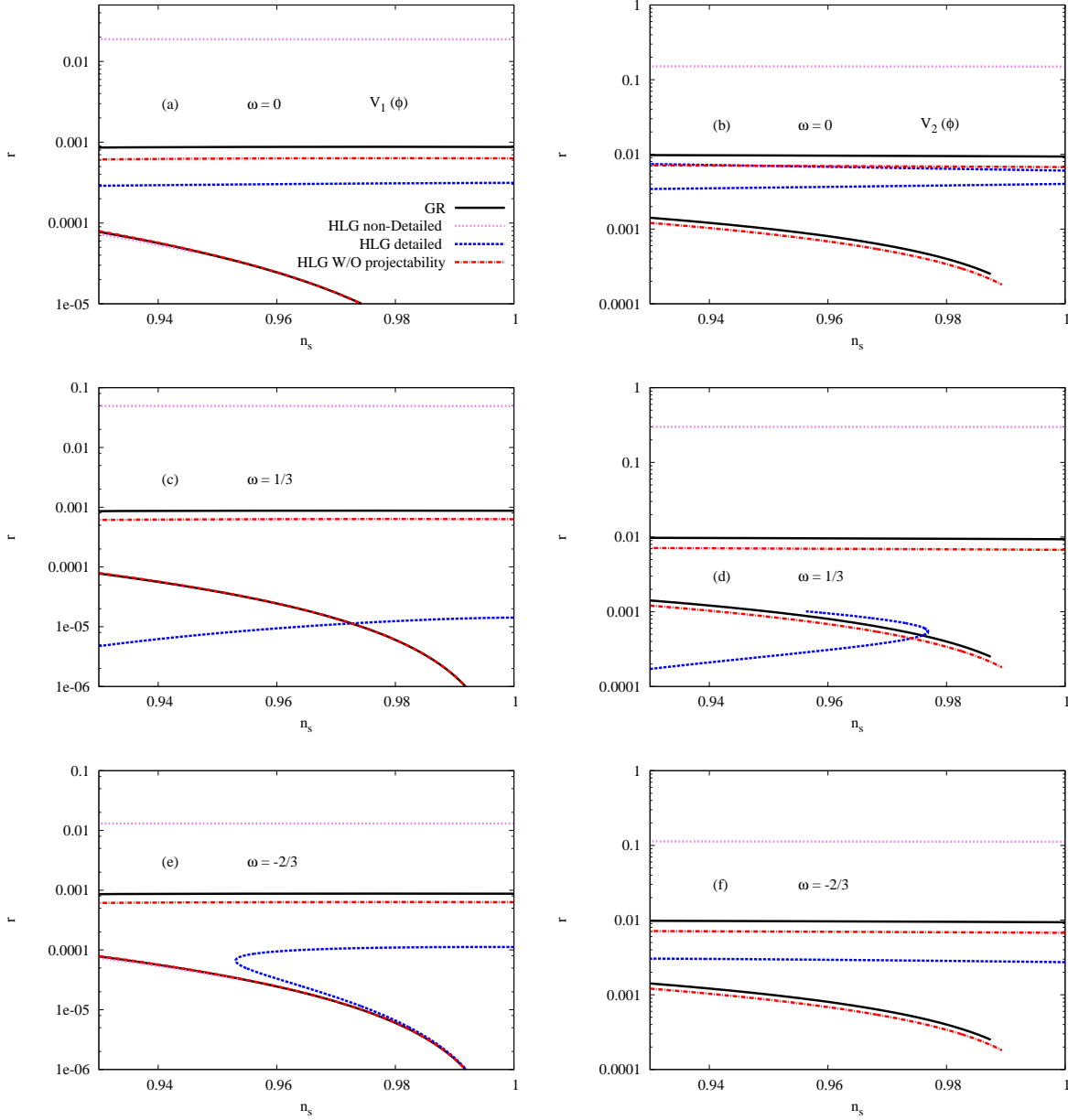


Fig. 6: The same as Fig. 2 but for tensorial-to-spectral density fluctuations (r) as functions of the spectral index (n_s), Eq. (40).

gravity).

We find that the dependence of the tensorial density fluctuations (P_t) on the inflation field ϕ varies with:

- a) the proposed inflation potentials,
- b) the equations of state characterizing the cosmic background, and
- c) the types/theories of gravity.

It is obvious that negative P_t means that the second term in Eq. (35) should be positive or Λ ranges between ϵH and $(\pi/2) H$, where ϵ is an infinitesimal positive constant.

Fig. 3 shows the same as in Fig. 2, but for spectral density fluctuations (P_s), Eq. (36). Also here, same conclusions can be drawn. Negative P_s refers to $\epsilon H \leq \Lambda \leq (\pi/2) H$ and furthermore, $\phi \leq 0$ leads to dominant

contribution from the cosmological constant.

Also here, P_s obtained from GR are almost equivalent to that from HL gravity without the projectibility condition. P_s from detailed HL gravity is the largest, except at $\omega = -2/3$, it becomes the smallest for $V_2(\phi)$ and second largest for $V_1(\phi)$. P_s from non-detailed HL gravity is almost a vanishing constant.

Fig. 4 presents the ratio of tensorial to spectral density fluctuations ($r = P_t/P_s$), Eq. (37) as functions of n_s . Again, the inflation fields, the equations of state, and the types of gravity apparently affect the obtained results. It is obvious that eliminating the dependence on various parameters through r seems to bring regularities to all curves. Regularity means concrete patterns among both potentials and the equations of state. The largest r -results are obtained from non-detailed condition, while detailed condition gives the smallest results. The results from GR and HL gravity without the projectibility condition are similar at least qualitatively, where the earlier lead to larger values than the latter.

In order to determine the role of HL relative to GR gravity, we draw in Fig. 5 Δr which is the differences between HL and GR on r as a function of ϕ/M_p from both scalar fields and the three types of the equation of state. The GR-HL results are given as dashed-lines with diamonds (non-detailed HL), dotted lines with squares (HL without projectibility) and double-dotted dashed lines with triangles (detailed HL)]. For a better comparison, we also illustrate the pure GR contributions as solid curves.

When $V_1(\phi)$ is taken into consideration, it is obvious that Δr is almost identical for all equations of state. Furthermore, their values are very close to that of pure GR gravity. This means that HL gravity comes up with negligibly small contributions, especially at $\omega = 0$. At $\omega = 1/3$ and $-2/3$, we find that HL results on Δr become faster than that of the pure GR gravity, especially when the scalar fields range between 10% to $\sim 25\%$ of M_p .

For $V_2(\phi)$, there is a noticeable differ between Δr from non-detailed HL and pure GR gravity. The rate of Δr decreasing with increasing ϕ/M_p becomes faster relative the all other results. At large ϕ/M_p , the contributions from the non-detailed HL gravity become dominant (negative Δr).

When the cosmic background geometry is characterized by $\omega = 0$, HL has a tiny impact, except for non-detailed condition in $V_2(\phi)$. Apart from the remarkable impacts of non-detailed HL gravity, the differences between GR and other HL theories become noticeable at $0.1 < \phi < 0.25 M_p$ for $\omega = 1/3$ and $-2/3$.

C. Recent PLANCK Observations

In Fig. 6, the tensorial-to-spectral density fluctuations ratio (r) are depicted as functions of the spectral index (n_s). Recently, such a dependence has been analysed by PLANCK collaboration, Fig. 7, as well.

In Fig. 7, some of the results deduced from the parametric dependence of r on the spectral index (n_s) are confronted to the recent PLANCK. Obviously, they fit well with the observations at 68% and 95% confidence level [1, 2]. The figure also illustrates a significant improvement (red and blue contours) with respect to previous PLANCK data release (gray contours).

The recent PLANCK observations set an upper bound to the tensor-to-scalar ratio, $r_{0.002} < 0.11$ at 95% confidence level, when taking into consideration the PLANCK high- ℓ polarization data. The upper limit according to B -mode polarization constraint, $r < 0.12$ at 95 % at confidence level, which was obtained from a joint analysis of PLANCK, BICEP2, and Keck Array data [1] is well reproduced.

In Fig. 7, we draw the largest r as obtained from our parametric calculations (solid line). This was deduced from non-detailed HL gravity. As shown in Fig. 5, only non-detailed HL gravity causes a noticeable impact relative to the pure GR gravity. We depict largest r calculated from $V_1(\phi)$ as the bottom three lines; at $\omega = 1/3$ (solid), $\omega = 0$ (dashed) and $\omega = -2/3$ (dotted line). The small values can be interpreted due to the negligibly small Δr , Fig. 5. The results from $V_2(\phi)$ are presented by the top three lines; at $\omega = 1/3$ (double dotted), $\omega = 0$ (long dashed) and $\omega = -2/3$ (dot-dashed line). It is obvious that almost all our calculations fit well with the recent observations, except from $V_2(\phi)$ at $\omega = 1/3$ (double dotted). Remaining results on r vs. n_s agree well with the recent observations. Among other theories of gravity, it is obvious that non-detailed HL gravity results in the largest r , e.g. 1 – 2 orders of magnitudes.

IV. CONCLUSIONS

Based on the standard model of elementary particles, i.e. the successful description of all forces (except gravity) by quantum field theory, and Lifshitz theory with an anisotropic scaling in UV between Minkowskian

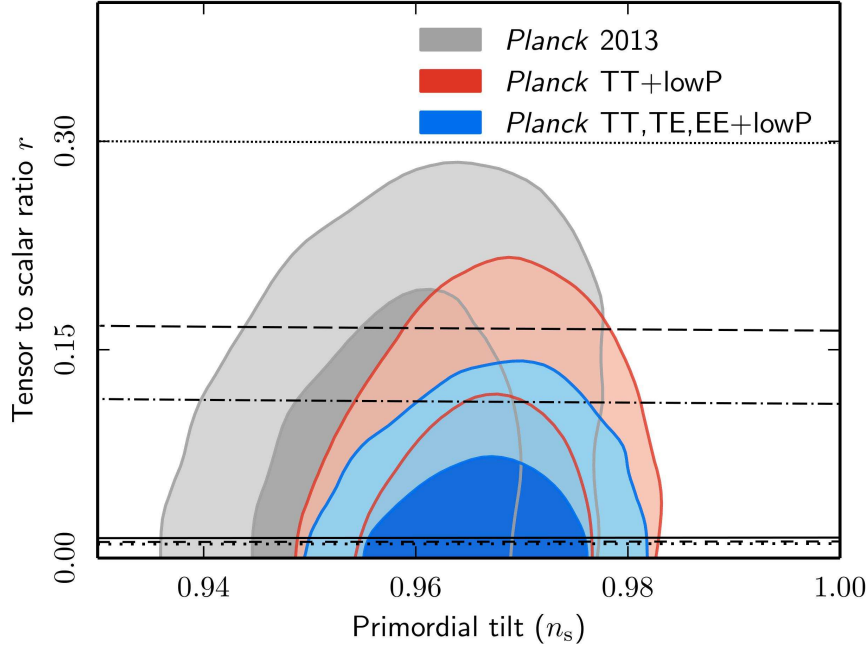


Fig. 7: Some results illustrated in Fig. 6 are confronted here to the recent PLANCK observations (red and blue contours), which is also compared to previous observations (gray contours). The horizontal lines represents largest r obtained through parametric calculations from $V_1(\phi)$ (bottom three lines) with non-detailed HL gravity at $\omega = 1/3$ (solid), $\omega = 0$ (dashed) and $\omega = -2/3$ (dotted line). The results from $V_2(\phi)$ (top three lines) with non-detailed HL gravity at $\omega = 1/3$ (double dotted), $\omega = 0$ (long dashed) and $\omega = -2/3$ (dot-dashed line).

space and time which was based on a scalar field theory for condensed matter physics, Horava proposed a theory for quantum gravity with an anisotropic scaling in UV. He even concluded that such modification in gravity is responsible for the accelerated expansion of the Universe. In GR and various approaches for HL gravity, we have studied the impacts of different equations-of-state on Friedmann inflation with a scalar field. HL gravity possesses power-counting renormalizability and assures causal dynamical triangulations. Its renormalization is based on asymptotic safety and symmetries of GR. HL gravity introduces a new set of symmetries imposing invariance under foliation-preserving diffeomorphisms.

The present work describes a systematic analysis for the effects of different equations-of-state characterizing the cosmic background geometry and compares between the conventional gravity characterized by GR and its HL counterparts; detailed, and non-detailed HL gravity, and that without the projectability condition.

We have studied the dependence of tensorial and spectral density fluctuations on two scalar fields, $V_1(\phi)$ [Eq. (30) and $V_2(\phi)$ Eq. (32)]. The latter are imposed to assure inflationary mechanism. It was found that the proposed inflation potentials, and/or the equations of state characterizing the cosmic background geometry, and/or gravity theories control how density fluctuations vary with the scalar field. We found that the GR results on tensorial (P_t) and spectral density fluctuations (P_s) are almost equivalent to HL gravity without the projectability condition, at least qualitatively. Also we notice that $V_1(\phi)$ results in values smaller than the ones deduced from $V_2(\phi)$. The results on both P_t and P_s from detailed and non-detailed HL gravity strongly depend on the equation of state and of course on the inflation potential field.

The dependence of the tensorial-to-spectral density fluctuations (r) on the quantum perturbations in form of scalar spectra index has been calculated. There is a regular pattern characterizing both potentials and the equations of state. The largest results are obtained from non-detailed HL gravity, while detailed condition gives the smallest results. The results from GR and HL gravity without the projectability condition are almost identical. The earlier lead to larger values than the latter.

We conclude that negative tensorial fluctuations refer to dominant contributions by the cosmological constant. Negative spectral fluctuations summarize that effect plus that from scalar field temporal evolution. Also, we find that the tensorial-to-spectral density fluctuations contentiously decrease when moving from non-detailed

HL gravity, to GR gravity, to HL gravity without the projectibility condition, and to detailed HL gravity. Last but not least, we find that GR results are very close to that from HL gravity without the projectibility condition.

Our calculations on the tensorial-to-spectral density fluctuations are compared with the recent PLANCK observations, i.e. $r_{0.002} < 0.11$ at 95% confidence level in PLANCK high- ℓ polarization data. The upper limit according to B -mode polarization constrains r to < 0.12 at 95 % at confidence level. The latter was obtained from a joint analysis of PLANCK, BICEP2, and Keck Array data [1]. It is obvious that our parametric calculations agree well with the recent PLANCK observations.

At $\omega = 0$, HL gravities have tiny impacts, except HL with non-detailed condition in scalar field $V_2(\phi)$. Apart from the dominant role of non-detailed HL relative to GR gravity, the differences between GR and other HL theories are only distinguishable from the pure GR at $0.1 < \phi < 0.25 M_p$ for $\omega = 1/3$ and $-2/3$.

Last but not least, it seems that introducing *extra* degrees of freedom in forms of additional scalar fields which were assumed in order to derive the inflation, enables us to describe well the recent PLACK observations.

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